Priprema za treći pismeni ispit (Potencije i algebarski izrazi)

1. Izračunaj:

a) $(0.2)^{-4}\cdot(-1.6) = \left(\frac{2}{10}\right)^{-4}\cdot\left(-\frac{16}{10}\right) = \left(\frac{1}{5}\right)^{-4}\cdot\left(-\frac{8}{5}\right) = 5^4\cdot\left(-\frac{8}{5}\right) = 125\cdot(-8) = -1000$

b) $\left[6-4\cdot\left(\frac{5}{16}\right)^0\right]^{-2} = \left[6-4\cdot 1\right]^{-2} = \left[6-4\right]^{-2} = 2^{-2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

c) $\frac{2^3\cdot 3^{-2}}{6^{-1}} = \frac{2^3\cdot 6^1}{3^2} = \frac{8\cdot 6}{9_3} = \frac{16}{3}$

d) $\frac{1}{32}\cdot 8^5\cdot 0.25^{-2} = \frac{1}{32}\cdot 8^5\cdot\left(\frac{25}{100}\right)^{-2} = 2^{-5}\cdot\left(2^3\right)^5\cdot\left(2^{-2}\right)^{-2} = 2^{-5}\cdot 2^{15}\cdot 2^4 = 2^{14} = 16384$

2. Zapiši u obliku potencije s bazom 10:

a) sto milijardi $100\,000\,000\,000 = 10^{11}$

b) $0.0000001 = \frac{1}{10000000} = 10^{-7}$

c) $0.001\cdot\frac{10^9}{10000}\cdot 0.00001 : 10^{-7} = 10^{-3}\cdot\frac{10^9}{10^4}\cdot 10^{-5} : 10^{-7} = 10^{-3}\cdot 10^5\cdot 10^{-5} : 10^{-7} =$
$= 10^{-3} : 10^{-7} = 10^{-3-(-7)} = 10^{-3+7} = 10^4$

d) $\left(5^{31}\cdot 4^{15}\right):\left(5^{11}\cdot 4^5\right) = 5^{20}\cdot 4^{10} = 5^{20}\cdot\left(2^2\right)^{10} = 5^{20}\cdot 2^{20} = 10^{20}$

3. Zapiši u obliku potencije s bazom 3:

a) $5\cdot 9^5+12\cdot 3^9 = 5\cdot\left(3^2\right)^5 + 2^2\cdot 3\cdot 3^9 = 5\cdot 3^{10} + 4\cdot 3^{10} = 9\cdot 3^{10} = 3^2\cdot 3^{10} = 3^{12}$

b) $\left(-3^2\right)^3+5\cdot(-3)^6-\left(-3^3\right)^2 = -3^6+5\cdot 3^6-3^6 = 3^6(-1+5-1) = 3^6\cdot 3 = 3^7$

c) $\left(27^2\cdot 81\cdot 9^3\right)^4 = \left(\left(3^3\right)^2\cdot 3^4\cdot\left(3^2\right)^3\right)^4 = \left(3^6\cdot 3^4\cdot 3^6\right)^4 = \left(3^{16}\right)^4 = 3^{64}$

d) $9^{3n}:27^{-2n-3} = \left(3^2\right)^{3n} : \left(3^3\right)^{-2n-3} = 3^{6n} : 3^{-6n-9} = 3^{6n-(-6n-9)} =$
$= 3^{6n+6n+9} = 3^{12n+9}$

4. Poredaj po veličini brojeve 10^5 , $(-10)^5$, 10^{-5} i $(-10)^{-5}$.

$10^5 = 100000$ (4.), $(-10)^5 = -100000$ (1.), $10^{-5} = \left(\frac{1}{10}\right)^5 = \frac{1}{100000}$ (3.), $(-10)^{-5} = \left(-\frac{1}{10}\right)^5 = -\frac{1}{100000}$ (2.)

$(-10)^5 < (-10)^{-5} < 10^{-5} < 10^5$

5. Pojednostavni:

a) $(-a^2)^3+(-a^3)^2+((-a)^{-2})^{-3} = -a^6+a^6+(-a)^6 = a^6$

b) $(125a^5b^{-3})^4:(25a^{10}b^{-7})^{-6} = (5^3a^5b^{-3})^4 : (5^2a^{10}b^{-7})^{-6} = 5^{12}a^{20}b^{-12} : 5^{-12}a^{-60}b^{42} = 5^{24}a^{80}b^{-54} = \frac{5^{24}a^{80}}{b^{54}}$

c) $\left(\frac{2a^{15}}{7b^{-4}}\right)^{-5}\cdot\left(\frac{49b^4}{8a^{-9}}\right)^6 = \left(\frac{7^2}{2^3}\right)^{-5}\cdot\left(\frac{7^2b^4}{2^3a^{-9}}\right)^6 = \frac{2^{-5}a^{-75}}{7^{-5}b^{20}}\cdot\frac{7^{12}b^{24}}{2^{18}a^{-54}} = \frac{7^5}{2^5a^{-21}}\cdot\frac{7^{12}a^{54}b^{24}}{2^{18}} = \frac{7^{17}b^4}{2^{23}a^{21}}$

d) $\left[\frac{3}{10}(x^2)^{16}-\frac{3}{5}(x^3)^{12}\right]:\frac{3}{5}(x^7)^3 = \left[\frac{3}{10}x^{32}-\frac{3}{5}x^{36}\right]:\frac{3}{5}x^{21} = \frac{1}{2}x^{11}-x^{15}$

$\frac{3}{10}:\frac{3}{5} = \frac{3}{10}\cdot\frac{5}{3} = \frac{1}{2}$

6. Zapiši u znanstvenom zapisu:

a) $(0.000328)^2 = (3,28\cdot10^{-4})^2 = 10,7584\cdot10^{-8} = 1,07584\cdot10^{-9}$

b) $\frac{-3.2\cdot10^{-8}}{8\cdot10^{-6}} = \frac{-3,2\cdot10^{-8}}{8\cdot10^{-6}} = -\frac{0,4}{10^2} = -0,004 = -4\cdot10^{-3}$

c) masu virusa gripe koja iznosi $707\cdot10^{-15}$ kg $7,07\cdot10^2\cdot10^{-15} = 7,07\cdot10^{-13}$

d) starost Zemlje koja iznosi 4600 milijuna godina $4600000000 = 4,6\cdot10^9$

7. Mravlji mozak ima $2.5\cdot10^5$ moždanih stanica, a ljudski 10^{11} . Koliko mrava ima moždanih stanica kao jedan čovjek? Rješenje napiši u znanstvenom zapisu.

$10^{11}:(2,5\cdot10^5) = 0,4\cdot10^6 = 4\cdot10^{-1}\cdot10^6 = 4\cdot10^5$

$1:2,5 = 1:\frac{25}{10} = \frac{1}{1}\cdot\frac{2}{5} = \frac{2}{5} = 0,4$

8. Prosječna masa čovjeka je 70 kg. Plavetni kit ima masu kao 2667 ljudi, a 700000 pčela kao jedan čovjek. Koliko pčela ima masu kao jedan plavetni kit? Rješenje napiši u znanstvenom zapisu.

$2667\cdot700000 = 1866900000 = 1,8669\cdot10^9$

9. Izvedi pretvorbe:

a) 0.7 cm = $0,7\cdot10^{-2}\cdot10^{-3} = 0,7\cdot10^{-5} = 7\cdot10^{-6} = 0,000007$ km

b) 2 m 7 cm 5 mm = $2,075\cdot10^6 = 2075000$ µm

c) 0.005 m^2 = $0,005\cdot(10^2)^2 = 0,005\cdot10^4 = 5\cdot10^{-3}\cdot10^4 = 5\cdot10^1 = 50$ cm^2

d) 200 ml = $200\cdot10^{-3}\cdot10^{-2} = 200\cdot10^{-5} = 2\cdot10^{-3} = 0,002$ hl

e) 8 dana 11h = $203\cdot3600 = 730200 = 7,302\cdot10^5$ s

f) 5.2 TB = $5,2\cdot10^6 = 5200000$ MB