Ponavljanje za pismeni ispit – korijeni

1. Izračunaj:
   a. $4\sqrt{\frac{36}{49}}-\frac{1}{7}\sqrt{\frac{81}{25}}$,

   $4\sqrt{\frac{36}{49}}-\frac{1}{7}\sqrt{\frac{81}{25}}=4\cdot\frac{6}{7}-\frac{1}{7}\cdot\frac{9}{5}=\frac{24}{7}-\frac{9}{35}=\frac{120-9}{35}=\boxed{\frac{111}{35}}$

   b. $3\sqrt{0.25}-4\sqrt{0.0081}$,

   $3\sqrt{0.25}-4\sqrt{0.0081}=3\cdot 0.5-4\cdot 0.09=1.5-0.36=\boxed{1.14}$

   c. $\sqrt[3]{0.027}-2\sqrt[3]{0.008}$.

   $\sqrt[3]{0.027}-2\sqrt[3]{0.008}=0.3-2\cdot 0.2=0.3-0.4=\boxed{-0.1}$

2. Djelomično korjenuj:
   a. $\sqrt{\frac{12a^7b^3}{c^5}}$,

   $\sqrt{\frac{12a^7b^3}{c^5}}=\sqrt{\frac{4\cdot 3\cdot a^6\cdot a\cdot b^2\cdot b}{c^4\cdot c}}=\boxed{\frac{2a^3b}{c^2}\sqrt{\frac{3ab}{c}}}$

   b. $\sqrt[3]{\frac{54x^6y^5}{z^4}}$.

   $\sqrt[3]{\frac{54x^6y^5}{z^4}}=\sqrt[3]{\frac{27\cdot 2\cdot x^6\cdot y^3\cdot y^2}{z^3\cdot z}}=\boxed{\frac{3x^2y}{z}\sqrt[3]{\frac{2y^2}{z}}}$

3. Unesi pod znak korijena:
   a. $a^3\sqrt{a}$,

   $a^3\sqrt{a}=\sqrt{\left(a^3\right)^2\cdot a}=\sqrt{a^6\cdot a}=\boxed{\sqrt{a^7}}$

   b. $a^3\sqrt{a}$.

   $x\sqrt[3]{x^2}=\sqrt[3]{x^3\cdot x^2}=\boxed{\sqrt[3]{x^5}}$

4. Izračunaj:
   a. $4\sqrt{160}-7\sqrt{810}+\sqrt{4000}$,

   $4\sqrt{160}-7\sqrt{810}+\sqrt{4000}=4\sqrt{16\cdot 10}-7\sqrt{81\cdot 10}+\sqrt{400\cdot 10}=$
   $4\cdot 4\sqrt{10}-7\cdot 9\sqrt{10}+20\sqrt{10}=16\sqrt{10}-63\sqrt{10}+20\sqrt{10}=\boxed{-27\sqrt{10}}$

   b. $12\sqrt[3]{54}-3\sqrt[3]{16}+\sqrt[3]{250}$.

$12\sqrt[3]{54}-3\sqrt[3]{16}+\sqrt[3]{250}=12\sqrt[3]{27\cdot 2}-3\sqrt[3]{8\cdot 2}+\sqrt[3]{125\cdot 2}=12\cdot 3\sqrt[3]{2}-3\cdot 2\sqrt[3]{2}+5\sqrt[3]{2}=$
$36\sqrt[3]{2}-6\sqrt[3]{2}+5\sqrt[3]{2}=\boxed{35\sqrt[3]{2}}$

5. Izračunaj:

a. $\sqrt{72}-\left(\sqrt{6}-2\sqrt{3}\right)^2$,

$\sqrt{72}-\left(\sqrt{6}-2\sqrt{3}\right)^2$ = kvadrat razlike

$=\sqrt{72}-\left(\left(\sqrt{6}\right)^2-2\cdot\sqrt{6}\cdot 2\sqrt{3}+\left(2\sqrt{3}\right)^2\right)=\sqrt{72}-\left(6-4\sqrt{18}+4\cdot 3\right)=$

$\sqrt{72}-\left(6-4\sqrt{18}+12\right)=\sqrt{72}-\left(18-4\sqrt{18}\right)=\sqrt{72}-18+4\sqrt{18}=$

$\sqrt{36\cdot 2}-18+4\sqrt{9\cdot 2}=6\sqrt{2}-18+4\cdot 3\sqrt{2}=6\sqrt{2}-18+12\sqrt{2}=\boxed{18\sqrt{2}-18}$

b. $\left(\sqrt[3]{\frac{x^2}{y^2}}-\sqrt[3]{\frac{x}{y}}+1\right)\left(\sqrt[3]{\frac{x}{y}}+1\right)$,

$\left(\sqrt[3]{\frac{x^2}{y^2}}-\sqrt[3]{\frac{x}{y}}+1\right)\left(\sqrt[3]{\frac{x}{y}}+1\right)=\left(\sqrt[3]{\frac{x}{y}}+1\right)\left(\sqrt[3]{\frac{x^2}{y^2}}-\sqrt[3]{\frac{x}{y}}+1\right)$ = zbroj kubova

$=\left(\sqrt[3]{\frac{x}{y}}\right)^3+1^3=\boxed{\frac{x}{y}+1}$

6. Racionaliziraj:

a. $\frac{2}{\sqrt[3]{2}}$,

$\frac{2}{\sqrt[3]{2}}=\frac{2}{\sqrt[3]{2}}\cdot\frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}=\frac{2\sqrt[3]{2^2}}{\sqrt[3]{2^3}}=\frac{2\sqrt[3]{2^2}}{2}=\sqrt[3]{2^2}=\boxed{\sqrt[3]{4}}$

b. $\frac{2\sqrt{3}-\sqrt{2}}{2\sqrt{3}+\sqrt{2}}$,

$\frac{2\sqrt{3}-\sqrt{2}}{2\sqrt{3}+\sqrt{2}}=\frac{2\sqrt{3}-\sqrt{2}}{2\sqrt{3}+\sqrt{2}}\cdot\frac{2\sqrt{3}-\sqrt{2}}{2\sqrt{3}-\sqrt{2}}=\frac{\left(2\sqrt{3}-\sqrt{2}\right)^2}{\left(2\sqrt{3}+\sqrt{2}\right)\left(2\sqrt{3}-\sqrt{2}\right)}$ = u brojniku je kvadrat razlike, a u nazivniku razlika kvadrata

$=\frac{\left(2\sqrt{3}\right)^2-2\cdot 2\sqrt{3}\cdot\sqrt{2}+\left(\sqrt{2}\right)^2}{\left(2\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}=\frac{4\cdot 3-4\sqrt{6}+2}{4\cdot 3-2}=\frac{12-4\sqrt{6}+2}{12-2}=\frac{14-4\sqrt{6}}{10}=$

$\frac{2\left(7-2\sqrt{6}\right)}{10}=\boxed{\frac{7-2\sqrt{6}}{5}}$

7. Zlatar je 27 zlatnih kockica s bridom duljine 2 mm pretopio u jednu kocku. Kolika je duljina brida nove kocke?

Obujam nove kocke mora biti jednak zbroju obujama 27 malih kocaka.

$V_m=2^3\ \text{mm}^3=8\ \text{mm}^3$

$V_v=27\cdot 8\ \text{mm}^3=216\ \text{mm}^3$

$V_v = a^3$

$a^3 = 216$

$a = \sqrt[3]{216}$

$a = 6\,\text{mm}$

Duljina brida nove kocke iznosi 6 mm.