Ponavljanje za pismeni ispit – kompleksni brojevi

1. Odredi realne brojeve x i y za koje vrijedi jednakost $(x+yi)(3+i)=-9+7i$.

$(x+yi)(3+i)=-9+7i$
$3x+xi+3yi+yi^2=-9+7i$
$3x+xi+3yi-y=-9+7i$
$3x-y+i(x+3y)=-9+7i$
$3x-y=-9$
$x+3y=7/\cdot(-3)$
$3x-y=-9$
$-3x-9y=-21$
$-10y=-30$
$\boxed{y=3}$
$x+9=7$
$\boxed{x=-2}$

2. Odredi realne brojeve m i n takve da brojevi $z_1=(2m-n)-i(n-4)$ i $z_2=(8m-1)+i(4m+3n)$ budu kompleksno konjugirani.

$\mathrm{Re}\,z_1=\mathrm{Re}\,z_2$
$\mathrm{Im}\,z_1=-\mathrm{Im}\,z_2$
$2m-n=8m-1$
$-n+4=-4m-3n$
$2m-n-8m=-1$
$-n+4m+3n=-4$
$-6m-n=-1/\cdot 2$
$4m+2n=-4$
$-12m-2n=-2$
$4m+2n=-4$
$-8m=-6$
$m=\frac{6}{8}$
$\boxed{m=\frac{3}{4}}$
$-6m-n=-1$
$-6\cdot\frac{3}{4}-n=-1$
$-\frac{18}{4}-n=-1$
$-\frac{9}{2}-n=-1$
$-n=-1+\frac{9}{2}$
$-n=\frac{7}{2}$
$\boxed{n=-\frac{7}{2}}$

3. Izračunaj $\sqrt{-25}+3\sqrt{-12}-7\sqrt{-144}-\sqrt{-48}$.

$\sqrt{-25}+3\sqrt{-12}-7\sqrt{-144}-\sqrt{-48}=$
$\sqrt{25\cdot(-1)}+3\sqrt{4\cdot3\cdot(-1)}-7\sqrt{144\cdot(-1)}-\sqrt{16\cdot3\cdot(-1)}=$
$5i+3\cdot2i\sqrt{3}-7\cdot12i-4i\sqrt{3}=$
$5i+6i\sqrt{3}-84i-4i\sqrt{3}=$
$\boxed{-79i+2i\sqrt{3}}$

4. Izračunaj $2i^{29}+i^3+12i^{191}-6i^{79}$.

$2i^{29}+i^3+12i^{191}-6i^{79}=2i^{4\cdot7+1}+i^3+12^{4\cdot47+3}-6i^{4\cdot19+3}=2i^1+i^3+12i^3-6i^3=$
$2i-i-12i+6i=\boxed{-5i}$

5. Rastavi izraz x^2+49 na kompleksne faktore.

$x^2+49=x^2+7^2=\boxed{(x+7i)(x-7i)}$

6. Rastavi broj 34 na kompleksne faktore.

$34=25+9=5^2+3^2=\boxed{(5+3i)(5-3i)}$

7. Ako su $z_1=3-5i$ i $z_2=3i$, odredi:
   a. $4z_1-5z_2$,

$4z_1-5z_2=4(3-5i)-5\cdot3i=12-20i-15i=\boxed{12-35i}$

   b. z_1^3,

$z_1^3=(3-5i)^3=3^3-3\cdot3^2\cdot5i+3\cdot3\cdot(5i)^2-(5i)^3=27-3\cdot9\cdot5i+3\cdot3\cdot25i^2-125i^3$
$=27-135i+225i^2-125i^3=27-135i-225+125i=\boxed{-198-10i}$

   c. $\dfrac{\overline{z_1}}{z_1-2}$,

$\dfrac{\overline{z_1}}{z_1-2}=\dfrac{3+5i}{3-5i-2}=\dfrac{3+5i}{1-5i}=\dfrac{3+5i}{1-5i}\cdot\dfrac{1+5i}{1+5i}=\dfrac{3+15i+5i+25i^2}{1^2+5^2}=\dfrac{3+20i-25}{1+25}=$
$\dfrac{-22+20i}{26}=\dfrac{-22}{26}+\dfrac{20i}{26}=\boxed{-\dfrac{11}{13}+\dfrac{10}{13}i}$

   d. $\dfrac{\overline{z_2^2}}{2i+z_2^2}$.

$\dfrac{\overline{z_2^2}}{2i+z_2^2}=\dfrac{\overline{9i^2}}{2i+9i^2}=\dfrac{\overline{-9}}{2i-9}=\dfrac{-9}{-9+2i}=\dfrac{-9}{-9+2i}\cdot\dfrac{-9-2i}{-9-2i}=\dfrac{81+18i}{(-9)^2+2^2}=\dfrac{81+18i}{81+4}=$
$\dfrac{81+18i}{85}=\boxed{\dfrac{81}{85}+\dfrac{18}{85}i}$

8. Izračunaj $\left(i^{201}-\frac{i^{306}}{i^{307}}\right)^{402}$.

$$\left(i^{201}-\frac{i^{306}}{i^{307}}\right)^{402}=\left(i^{4\cdot 50+1}-\frac{1}{i}\right)^{402}=\left(i-\frac{1}{i}\right)^{402}=\left(i-\frac{1}{i}\cdot\frac{i}{i}\right)^{402}=\left(i-\frac{i}{i^2}\right)^{402}=\left(i-\frac{i}{-1}\right)^{402}=$$
$$(i+i)^{402}=(2i)^{402}=2^{402}\cdot i^{402}=2^{402}\cdot i^{4\cdot 100+2}=2^{402}\cdot i^2=2^{402}\cdot(-1)=\boxed{-2^{402}}$$

9. Koliki je imaginarni dio broja $\frac{(3-4i)(1+i)}{2+3i}$?

$$z=\frac{(3-4i)(1+i)}{2+3i}=\frac{3+3i-4i-4i^2}{2+3i}=\frac{3-i+4}{2+3i}=\frac{7-i}{2+3i}=\frac{7-i}{2+3i}\cdot\frac{2-3i}{2-3i}=\frac{14-21i-2i+3i^2}{2^2+3^2}=$$
$$\frac{14-23i-3}{4+9}=\frac{11-23i}{13}=\frac{11}{13}-\frac{23}{13}i$$
$$\mathrm{Im}z=\boxed{-\frac{23}{13}}$$

10. Odredi $\mathrm{Re}\left(\frac{5-7i}{2+i}-3+i\right)$.

$$\mathrm{Re}\left(\frac{5-7i}{2+i}-3+i\right)=\mathrm{Re}\left(\frac{5-7i+(-3+i)(2+i)}{2+i}\right)=\mathrm{Re}\left(\frac{5-7i-6-3i+2i+i^2}{2+i}\right)=$$
$$\mathrm{Re}\left(\frac{-1-8i-1}{2+i}\right)=\mathrm{Re}\left(\frac{-2-8i}{2+i}\right)=\mathrm{Re}\left(\frac{-2-8i}{2+i}\cdot\frac{2-i}{2-i}\right)=\mathrm{Re}\left(\frac{-4+2i-16i+8i^2}{2^2+1^2}\right)=$$
$$\mathrm{Re}\left(\frac{-4-14i-8}{4+1}\right)=\mathrm{Re}\left(\frac{-12-14i}{5}\right)=\boxed{-\frac{12}{5}}$$

11. Izračunaj apsolutnu vrijednost broja $z=\frac{(1-2i)^6}{(-2+5i)^3}\cdot\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$.

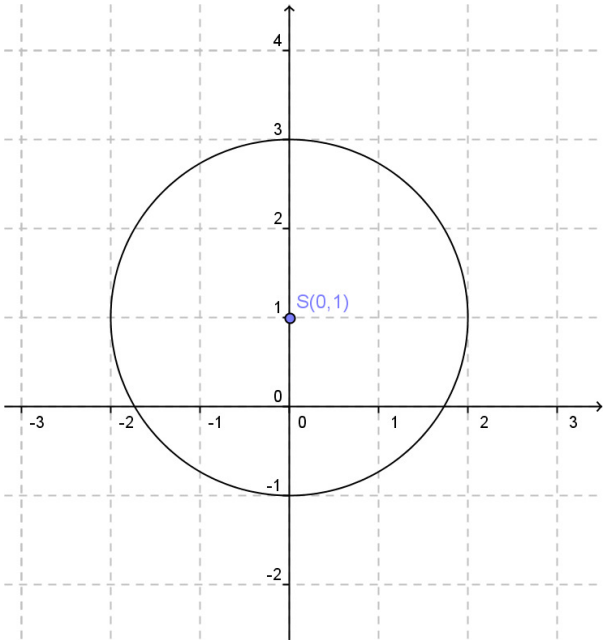
$$|z|=\left|\frac{(1-2i)^6}{(-2+5i)^3}\cdot\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)\right|=\frac{|1-2i|^6}{|-2+5i|^3}\cdot\left|\frac{1}{2}-\frac{\sqrt{3}}{2}i\right|=\frac{\sqrt{1^2+(-2)^2}^6}{\sqrt{(-2)^2+5^2}^3}\cdot\sqrt{\left(\frac{1}{2}\right)^2+\left(-\frac{\sqrt{3}}{2}\right)^2}=$$
$$\frac{\sqrt{1+4}^6}{\sqrt{4+25}^3}\cdot\sqrt{\frac{1}{4}+\frac{3}{4}}=\frac{\sqrt{5}^6}{\sqrt{29}^3}\cdot\sqrt{\frac{4}{4}}=\frac{5^3}{\sqrt{29^3}}\cdot\sqrt{1}=\frac{125}{\sqrt{29^2\cdot 29}}=\frac{125}{29\sqrt{29}}=\frac{125}{29\sqrt{29}}\cdot\frac{\sqrt{29}}{\sqrt{29}}=$$
$$\frac{125\sqrt{29}}{29\cdot 29}=\boxed{\frac{125\sqrt{29}}{841}}$$

12. Odredi i u Gaussovoj ravnini prikaži skup brojeva z za koje vrijedi uvjet $|z-i|=2$.

$|z-i|=2$
$z=x+yi$
$|x+yi-i|=2$
$|x+i(y-1)|=2$

$\sqrt{x^2+(y-1)^2}=2 \quad /^2$

$\boxed{x^2+(y-1)^2=4}$ - kružnica sa središtem u točki (0, 1) i polumjerom 2



13. Odredi i u Gaussovoj ravnini prikaži skup brojeva z za koje vrijedi uvjet $|z-i|=|z+2|$.

$|z-i|=|z+2|$

$z=x+yi$

$|x+yi-i|=|x+yi+2|$

$|x+i(y-1)|=|x+2+yi|$

$\sqrt{x^2+(y-1)^2}=\sqrt{(x+2)^2+y^2} \quad /^2$

$x^2+(y-1)^2=(x+2)^2+y^2$

$x^2+y^2-2y+1=x^2+4x+4+y^2$

$-2y=4x+4-1$

$-2y=4x+3 \quad /:(-2)$

$\boxed{y=-2x-\frac{3}{2}}$ - pravac