Ponavljanje za pismeni ispit – kvadratna jednadžba

1. Riješi jednadžbe:
   a. $0.25x^2+100=0$,

$\frac{25}{100}x^2+100=0$

$\frac{1}{4}x^2+100=0 \quad /\cdot 4$

$x^2+400=0$

$x^2=-400 \quad /\sqrt{\ }$

$\boxed{x_{1,2}=\pm 20i}$

   b. $(4x-1)^2-(3x+1)^2=0$,

$(4x)^2-2\cdot 4x\cdot 1+1^2-\left[(3x)^2+2\cdot 3x\cdot 1+1^2\right]=0$

$16x^2-8x+1-\left[9x^2+6x+1\right]=0$

$16x^2-8x+1-9x^2-6x-1=0$

$7x^2-14x=0 \quad /:7$

$x^2-2x=0$

$x(x-2)=0$

$x=0 \qquad\qquad x-2=0$

$\boxed{x_1=0} \qquad\qquad \boxed{x_2=2}$

   c. $\frac{x-2}{x}+\frac{x}{x+2}=3$.

$\frac{x-2}{x}+\frac{x}{x+2}=3 \quad /\cdot x(x+2) \qquad x\neq 0 \quad x+2\neq 0$

$x\neq -2$

$(x-2)(x+2)+x^2=3x(x+2)$

$x^2-4+x^2=3x^2+6x$

$2x^2-4-3x^2-6x=0$

$-x^2-6x-4=0 \quad /:(-1)$

$x^2+6x+4=0$

$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-6\pm\sqrt{6^2-4\cdot 1\cdot 4}}{2\cdot 1}=\frac{-6\pm\sqrt{36-16}}{2}=\frac{-6\pm\sqrt{20}}{2}=$

$\frac{-6\pm\sqrt{4\cdot 5}}{2}=\frac{-6\pm 2\sqrt{5}}{2}=\frac{2\left(-3\pm\sqrt{5}\right)}{2}=-3\pm\sqrt{5}$

$\boxed{x_{1,2}=-3\pm\sqrt{5}}$

2. Za koji realni broj k jednadžba $x^2+2(k+2)x+k^2+2=0$ ima dvostruko rješenje?

$a=1, b=2(k+2), c=k^2+2$

$D=b^2-4ac$

$D=[2(k+2)]^2-4\cdot 1\cdot\left(k^2+2\right)=4(k+2)^2-4\left(k^2+2\right)=4\left(k^2+4k+4\right)-4k^2-8=$

$4k^2+16k+16-4k^2-8=16k+8$

$D=0$

$16k+8=0$

$16k=-8 \quad /:16$

$k=-\frac{8}{16}$

$\boxed{k=-\frac{1}{2}}$

3. Za koji realni broj m jednadžba $mx^2-4mx+4m-1=0$ nema realnih rješenja?

$a=m, b=-4m, c=4m-1$

$D=b^2-4ac$

$D=[-4m]^2-4\cdot m\cdot(4m-1)=16m^2-16m^2+4m=4m$

$D<0$

$4m<0 \quad /:4$

$m<0$

$\boxed{m\in\langle-\infty,0\rangle}$

4. Ne rješavajući jednadžbu $2x^2-3x-6=0$, izračunaj:

   a. $x_1^2x_2+x_1x_2^2$,

$x_1+x_2=-\frac{b}{a}=\frac{3}{2} \quad (*)$

$x_1\cdot x_2=\frac{c}{a}=\frac{-6}{2}=-3 \quad (**)$

$x_1^2x_2+x_1x_2^2=x_1x_2(x_1+x_2)=-3\cdot\frac{3}{2}=\boxed{-\frac{9}{2}}$

   b. $(x_1+x_2)^{-2}$,

$(x_1+x_2)^{-2}=\frac{1}{(x_1+x_2)^2}=(*)=\frac{1}{\left(\frac{3}{2}\right)^2}=\frac{1}{\frac{9}{4}}=\boxed{\frac{4}{9}}$

   c. $x_1^{-2}+x_2^{-2}$.

$x_1^{-2}+x_2^{-2}=\frac{1}{{x_1}^2}+\frac{1}{{x_2}^2}=\frac{{x_1}^2+{x_2}^2}{{x_1}^2{x_2}^2}=\frac{{x_1}^2+2x_1x_2+{x_2}^2-2x_1x_2}{(x_1x_2)^2}=$

$\frac{(x_1+x_2)^2-2x_1x_2}{(x_1x_2)^2}=(*)\text{ i }(**)=\frac{\left(\frac{3}{2}\right)^2-2(-3)}{(-3)^2}=\frac{\frac{9}{4}+6}{9}=\frac{\frac{9+24}{4}}{9}=\frac{\frac{33}{4}}{\frac{9}{1}}=\frac{\frac{11}{4}}{\frac{3}{1}}=\boxed{\frac{11}{12}}$

5. Sastavi kvadratnu jednadžbu s cjelobrojnim koeficijentima kojoj je jedno rješenje $\frac{4}{2+\mathrm{i}}$.

$$x_1 = \frac{4}{2+\mathrm{i}} = \frac{4}{2+\mathrm{i}} \cdot \frac{2-\mathrm{i}}{2-\mathrm{i}} = \frac{8-4\mathrm{i}}{4+1} = \frac{8-4\mathrm{i}}{5} = \frac{8}{5} - \frac{4}{5}\mathrm{i}$$

$$x_2 = \frac{8}{5} + \frac{4}{5}\mathrm{i}$$

$$x_1 + x_2 = \frac{8}{5} - \frac{4}{5}\mathrm{i} + \frac{8}{5} + \frac{4}{5}\mathrm{i} = \frac{16}{5}$$

$$x_1 \cdot x_2 = \left(\frac{8}{5} - \frac{4}{5}\mathrm{i}\right)\left(\frac{8}{5} + \frac{4}{5}\mathrm{i}\right) = \frac{64}{25} + \frac{16}{25} = \frac{80}{25} = \frac{16}{5}$$

$$x^2 - (x_1 + x_2)x + x_1 x_2 = 0$$

$$x^2 - \frac{16}{5}x + \frac{16}{5} = 0 \quad /\cdot 5$$

$$\boxed{5x^2 - 16x + 16 = 0}$$

6. Skrati razlomak $\frac{10x^2 - x - 3}{50x^2 - 18}$.

$$10x^2 - x - 3 = 0$$

$$x_{1,2} = \frac{1 \pm \sqrt{(-1)^2 - 4 \cdot 10 \cdot (-3)}}{2 \cdot 10} = \frac{1 \pm \sqrt{1+120}}{20} = \frac{1 \pm \sqrt{121}}{20} = \frac{1 \pm 11}{20}$$

$$x_1 = \frac{1-11}{20} = \frac{-10}{20} = -\frac{1}{2}$$

$$x_2 = \frac{1+11}{20} = \frac{12}{20} = \frac{3}{5}$$

$$\frac{10x^2 - x - 3}{50x^2 - 18} = \frac{10\left(x - \left(-\frac{1}{2}\right)\right)\left(x - \frac{3}{5}\right)}{2\left(25x^2 - 9\right)} = \frac{2 \cdot 5\left(x + \frac{1}{2}\right)\left(x - \frac{3}{5}\right)}{2(5x-3)(5x+3)} = \frac{(2x+1)(5x-3)}{2(5x-3)(5x+3)} =$$

$$\boxed{\frac{(2x+1)}{2(5x+3)}}$$

7. Riješi jednadžbe:

a. $\left(x^2 + 2x\right)^2 - 14\left(x^2 + 2x\right) - 15 = 0$,

$x^2 + 2x = t \quad (*)$

$t^2 - 14t - 15 = 0$

$$t_{1,2} = \frac{14 \pm \sqrt{(-14)^2 - 4 \cdot 1 \cdot (-15)}}{2 \cdot 1} = \frac{14 \pm \sqrt{196+60}}{2} = \frac{14 \pm \sqrt{256}}{2} = \frac{14 \pm 16}{2}$$

$$t_1 = \frac{14-16}{2} = \frac{-2}{2} = -1$$

$$t_2 = \frac{14+16}{2} = \frac{30}{2} = 15$$

povratak u (*)

$x^2 + 2x = -1$	$x^2 + 2x = 15$
$x^2 + 2x + 1 = 0$	$x^2 + 2x - 15 = 0$
...	...

$\boxed{x_{1,2} = -1}$ $\boxed{x_3 = -5, x_4 = 3}$

b. $\frac{y^4}{6} - \frac{4y^2}{3} + 2 = 0$,

$\frac{y^4}{6} - \frac{4y^2}{3} + 2 = 0 \quad / \cdot 6$

$y^4 - 8y^2 + 12 = 0$

$\left(y^2\right)^2 - 8y^2 + 12 = 0$

$y^2 = t \quad (*)$

$t^2 - 8t + 12 = 0$

$t_{1,2} = \frac{8 \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 12}}{2 \cdot 1} = \frac{8 \pm \sqrt{64 - 48}}{2} = \frac{8 \pm \sqrt{16}}{2} = \frac{8 \pm 4}{2}$

$t_1 = \frac{8 - 4}{2} = \frac{4}{2} = 2$

$t_2 = \frac{8 + 4}{2} = \frac{12}{2} = 6$

povratak u (*)

$y^2 = 2 \quad / \sqrt{\ }$ $\qquad$ $y^2 = 6 \quad / \sqrt{\ }$

$\boxed{y_{1,2} = \pm\sqrt{2}}$ $\qquad$ $\boxed{y_{3,4} = \pm\sqrt{6}}$

8. Riješi sustav $\begin{array}{l} x^2 + y^2 - 8x - 12y + 27 = 0 \\ \underline{x - 2y + 3 = 0} \end{array}$.

$x - 2y + 3 = 0$

$x = 2y - 3 \quad (*)$

uvrštavanje u prvu jednadžbu

$(2y - 3)^2 + y^2 - 8(2y - 3) - 12y + 27 = 0$

$4y^2 - 12y + 9 + y^2 - 16y + 24 - 12y + 27 = 0$

$5y^2 - 40y + 60 = 0 \quad / : 5$

$y^2 - 8y + 12 = 0$

$y_{1,2} = \frac{8 \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 12}}{2 \cdot 1} = \frac{8 \pm \sqrt{64 - 48}}{2} = \frac{8 \pm \sqrt{16}}{2} = \frac{8 \pm 4}{2}$

$y_1 = \frac{8 - 4}{2} = \frac{4}{2} = 2$

$y_2 = \frac{8 + 4}{2} = \frac{12}{2} = 6$

povratak u (*)

$x_1 = 2y_1 - 3$ $\qquad$ $x_2 = 2y_2 - 3$

$x_1 = 2 \cdot 2 - 3$ $\qquad$ $x_2 = 2 \cdot 6 - 3$

$x_1 = 1$ $\qquad$ $x_2 = 9$

$\boxed{(1, 2), (9, 6)}$